

# TRANSPORTATION ALGORITHM (MODI METHOD)

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# STEPS OF MODI METHOD (TRANSPORTATION ALGORITHM)

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- The steps to evaluate unoccupied cells are as follows :

## Step 1:

- For an initial basic feasible solution with  $m + n - 1$  occupied cells, calculate  $u_i$  and  $v_j$  for rows and columns. The initial solution can be obtained by any of the three methods
  - North-West Corner Method.
  - Least Cost Method, and
  - Vogel's Approximation (or Penalty) Method.
- To start with, any one of  $u_j$ 's or  $v_j$ 's is assigned the value zero. It is better to assign zero for a particular  $u_i$  or  $v_j$  where there are maximum number of allocation in a row or column respectively, as it will reduce arithmetic work considerably.

# Cont.....

Then complete the calculation of  $u_i$ 's and  $v_j$ 's for other rows and columns by using the relation

$$c_{ij} = u_i + v_j \text{ for all occupied cells } (i, j).$$

**Step 2** For unoccupied cells, calculate opportunity cost (the difference that indicates the per unit cost reduction that can be achieved by an allocation in the unoccupied cell) by using the relationship  $d_{ij} = c_{ij} - (u_i + v_j)$  for all  $i$  and  $j$ .

**Step 3** Examine sign of each  $d_{ij}$

- i) If  $d_{ij} > 0$ , then current basic feasible solution is optimal.
- ii) If  $d_{ij} = 0$ , then current basic feasible solution will remain unaffected but an alternative solution exists.
- iii) If one or more  $d_{ij} < 0$ , then an improved solution can be obtained by entering unoccupied cell  $(i, j)$  in the basis.

# Cont.....

- An occupied cell having the largest negative value of  $d_{ij}$  is chosen for entering into the solution mix (new transportation schedule).
- **Step 4:** Construct a closed-path (or loop) for the unoccupied cell with largest negative opportunity cost. Start the closed path with the selected unoccupied cell and mark a plus sign (+) in this cell, trace a path along the rows (or column) to an occupied cell, mark the corner with minus sign (-) and continue down the column (or row) to an occupied cell and mark the corner with plus (+) sign and minus sign (-) alternatively. Close the path back to the selected unoccupied cell.

# Cont.....

- **Step 5** Select the smallest quantity amongst the cell marked with minus sign on the corners of closed loop. Allocate this value to the selected unoccupied cell and add it to other occupied cells marked with plus sign and subtract it from the occupied cells marked with minus sign.
- **Step 6** Obtain a new improved solution by allocating units to the unoccupied cell according to Step 5 and calculate the new total transportation cost.
- **Step 7** Test the revised solution further for optimality. The procedure terminates when all  $d_{ij} \geq 0$  , for unoccupied cells.

# Solve the following Transportation Problem

		Destination					Supply
		D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	
Source	S <sub>1</sub>	5	8	6	6	3	8
	S <sub>2</sub>	4	7	7	6	5	5
	S <sub>3</sub>	8	4	6	6	4	9
	Demand	4	4	5	4	8	22
						25	

# Cont.....

- **Solution** : Since the total destination requirement of 25 units exceeds the total resource capacity of 22 by 3 units, the problem is unbalanced.
- The excess requirement is handled by adding a dummy source,  $S_{\text{excess}}$  with a capacity equal to 3 units. We use zero unit transportation cost to the dummy source. The modified transportation table is shown in Table

**Initial solution**

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	Supply	
S <sub>1</sub>	5	8	6 (5)	6	3 (3)	8	$u_1 = 0$
S <sub>2</sub>	4 (4)	7	7	6 (1)	5	5	$u_2 =$
S <sub>3</sub>	8	4 (4)	6	6	4 (5)	9	$u_3 = 1$
$S_{\text{excess}}$	0	0	0	0 (3)	0	3	← $u_4 =$
<b>Demand</b>	4	4	5	4	8	25	
	$v_1 =$	$v_2 =$	$v_3 =$	$v_4 =$	$v_5 =$		

# Cont.....

- The initial solution is obtained by using Vogel's approximation method as shown in above Table.
- Since the solution includes 7 occupied cells, therefore, the initial solution is degenerate.
- In order to remove degeneracy we assign  $\Delta$  to unoccupied cell ( $S_2, D_5$ ) which has minimum cost among unoccupied cells as shown in the following table

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	Supply	u <sub>i</sub>
S <sub>1</sub>	5 +3	8 +5	6 5 (-)	6 +2	3 3 (+)	8	u <sub>1</sub> = 0
S <sub>2</sub>	4 4 (+)	7 +2	7 -1	6 1 (+)	5 Δ (-)	5	u <sub>2</sub> = 2
S <sub>3</sub>	8 +5	4 4 (+)	6 -1	6 +1	4 5 (+)	9	u <sub>3</sub> = 1
S <sub>excess</sub>	0 +2	0 +1	0 (+)	0 3 (-)	0 -7	3	u <sub>4</sub> = -4
Demand	4	4	5	4	8	25	
v <sub>j</sub>	v <sub>1</sub> = 2	v <sub>2</sub> = 3	v <sub>3</sub> = 6	v <sub>4</sub> = 4	v <sub>5</sub> = 3		

# Cont.....

- Determine  $u_i$  and  $v_j$  for occupied cells as shown in above table. Since opportunity cost in the cell (Sexcess, D3) is largest negative, it must enter the basis and the cell ( $S_2$ ,  $D_5$ ) must leave the basis. The new solution is shown in following table.

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	Supply	$u_i$
S <sub>1</sub>	5 +3	8 +5	6 5 (-)	6 +2	3 3 (+)	8	$u_1 = 0$
S <sub>2</sub>	4 4 (+)	7 +4	7 +1	6 1 (+)	5 +2	5	$u_2 = 0$
S <sub>3</sub>	8 +3	4 4 (+)	6 -1	6 (+)	4 5 (-)	9	$u_3 = 1$
S <sub>excess</sub>	0 +2	0 +3	0 Δ (+)	0 3 (-)	0 +3	3	$u_4 = -6$
Demand	4	4	5	4	8	25	
$v_j$	$v_1 = 4$	$v_2 = 3$	$v_3 = 6$	$v_4 = 6$	$v_5 = 3$		

# Cont.....

- Repeat the procedure of testing optimality of the solutions given in the previous table. The optimal solution is shown in the following table

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	Supply
S <sub>1</sub>	5	8	6	6	3 8	8
S <sub>2</sub>	4 4	7	7	6 1	5	5
S <sub>3</sub>	8	4 4	6 2	6 3	4	9
S <sub>excess</sub>	0	0	0 3	6	0	3
Demand	4	4	5	4	8	25

- The minimum total transportation cost associated with this solution is
- Total cost =  $(3 \times 8 + 4 \times 4 + 6 \times 1 + 4 \times 4 + 6 \times 2 + 6 \times 3) \times 10 = \text{Rs. } 920/-$

Thank You